Topological Quantum Computation

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Outline

Introduction

What is TQC?

One of the **quantum computational models** among many others:

- Quantum circuit
- Adiabatic quantum computation
- Topological quantum computation
- One-way quantum computation
- Holonomic quantum computation
- ...

They are **equivalent** in computational power (i.e. all universal), however, they have different merits and drawbacks.

Why is TQC?

Topology:

In mathematics, topology is concerned with the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing.

TQC conducts computation through some topological quantities of quantum systems, which is naturally **fault-tolerant** to some kind of error.

How is TQC?

If exchanging two particles gives rise to an extra phase factor $e^{i\varphi}$, this phase factor must square to 1 since the system has undergone a trivial loop. Then $\varphi = 0$ for bosons or $\varphi = \pi$ for fermions.

$$\psi(C_1) = \psi(C_2) = \psi(C_0).$$

In 3D space, there is only **ONE** kind of loop for a particle to circulate around another particle.

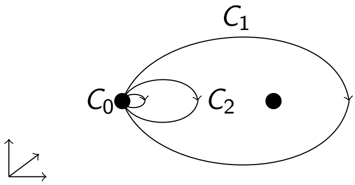

Figure: Loops for particle to circulate in 3D

How is TQC?

However, this is not the case in 2D space. C_1 can not continuously deform to C_2 without cutting.

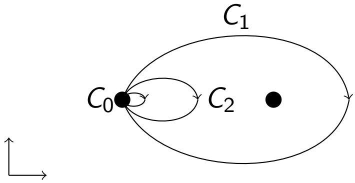

Figure: Loops for particle to circulate in 2D

Then it is possible to assign an arbitrary phase factor (**Abelian anyon**), or a unitray matrix (**non-Abelian anyon**):

$$\psi(C_1) = e^{i\varphi_a}\psi(C_2) \text{ or } \psi(C_1) = U\psi(C_2).$$

How is TQC?

TQC happens in the **exchange of anyons**. If the statistical evolutions are complex enough then they can realise arbitrary quantum algorithms.

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The problem is: 2D physical system does not exist.
Decouple two dimensions from the third one,

$$V(\boldsymbol{r}) = V_{xy}(x, y) + V_z(z) \;\; \Rightarrow \;\; \psi(\boldsymbol{r}) = \psi_{xy}(x, y)\psi_z(z).$$

Confinement along z direction can lead to energy gap, which protects the states from being excited (referring to adiabatic theorem). Now the system is essentially given by the 2D wave function $\psi_{xy}(x, y)$.

How is TQC?

Anyons in real life are not elementary particles, but **quasiparticles**.

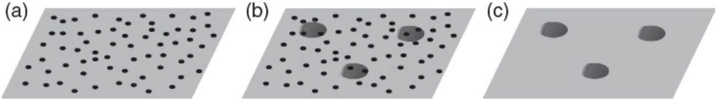


Figure: (a) A system with constituent particles confined on a plane that give rise to a 2D wave function. (b) Quasiparticles are identified as localised properties of the 2D wave function of the constituent particles. (c) Often we forget the constituent particles and we treat the quasiparticles as elementary ones living on the 2D space.

Heuristic Example: Aharonov-Bohm Effect

Result from quantum mechanics:
If a charged particle q adiabatically moves in a magnetic field described by vector potential $\boldsymbol{A}$, along a looping trajectory C, the wave function will acquire a phase:

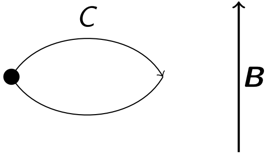


Figure: Charge in magnetic field

$$\varphi = \frac{q}{c\hbar} \oint_C \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{r} = \frac{q}{c\hbar} \oint_{S(C)} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = \frac{q}{c\hbar} \phi.$$

Heuristic Example: Aharonov-Bohm Effect

Now consider a infinitesimally thin solenoid with finite flux ϕ. The vector potential and magnetic field is given by

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\phi}{2\pi}\left(-\frac{y}{r^2}, \frac{x}{r^2}, 0\right)$$

and

$$\boldsymbol{B}(\boldsymbol{r}) = \phi\delta(r)\boldsymbol{e}_z.$$

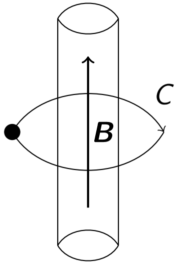


Figure: AB Effect

If the path surrounds $\boldsymbol{r}=0$, the phase attached to the wave function is then ($c=\hbar=1$ for simplicity)

$$\varphi = q\phi.$$

AB anyon: A **mechanical picture** of anyonic behavior. AB Anyon can be regarded as the composition of a charge and a solenoid.

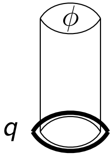
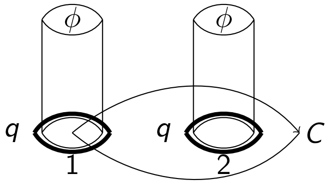
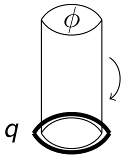


Figure: AB anyon

Heuristic Example: Aharonov-Bohm Effect

Circulate one AB anyon around another give rise to a phase of $2q\phi$. Indicate that the statistics phase of AB anyon is $q\phi$. Self-rotation of an AB anyon implies the spin of AB anyon is $\frac{q\phi}{2\pi}$.





$$\varphi = 2q\phi \Rightarrow \varphi_a = q\phi$$

$$\varphi = q\phi \Rightarrow s = \frac{q\phi}{2\pi}$$

Section Summary

- The foundation of TQC is anyon statistics. Due to the topological nature of anyon, TQC is robust to some kind of error.
- Anyons are **quasiparticles** arises as localised properties of effective 2D wave function.
- AB anyon is a picture for Abelian anyon. This is due to the $U(1)$ gauge invariance nature of electromagnetic field. We can envisage the AB effect in terms of non-Abelian charges and fluxes.
- Abelian anyon **cannot** form a universal set for quantum computation (only phase change). More interesting and useful part lies in non-Abelian anyon.

Quantum Double Model

Kitaev Toric Code

The origin of TQC:

Kitaev, 2003, *Fault-tolerant quantum computation by anyons*

In this paper, **toric code** is used to demonstrate the fault-tolerant nature of TQC. Toric Code, denoted by $D(Z_2)$ is the simplest **quantum double model**.

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Quantum double model $D(G)$:

Lattice realisations of topological systems. They are based on a finite group, G, that acts on spin states, defined on the links of the lattice.

Still, toric code can only support Abelian anyons. Though not universal, it can be used for quantum memory.

Kitaev Toric Code

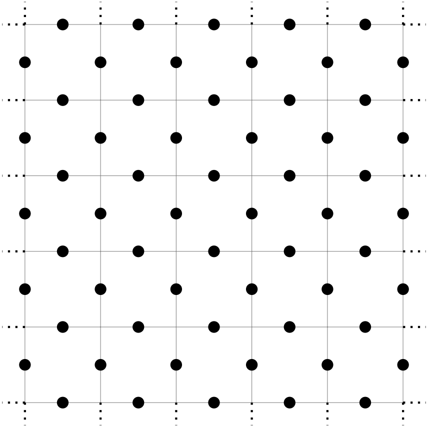
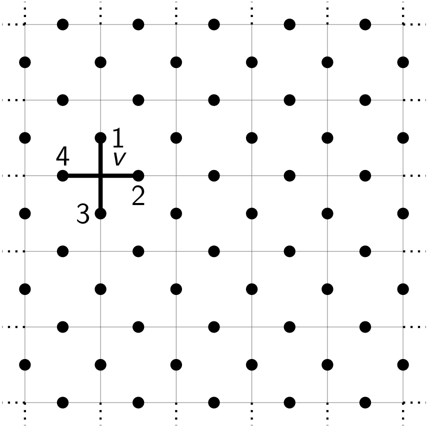


Figure: Lattice of Kitaev toric code

Kitaev Toric Code



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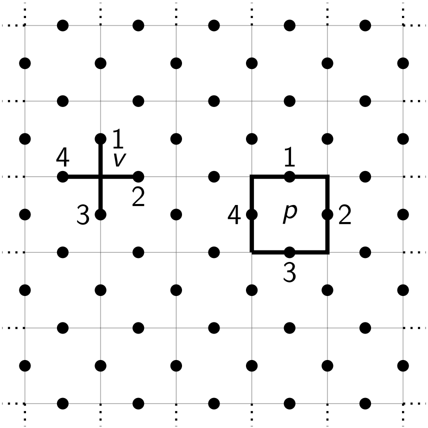

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Kitaev Toric Code

Vertex term:

$$A(v) = \sigma^x_{v,1}\sigma^x_{v,2}\sigma^x_{v,3}\sigma^x_{v,4}$$

Plaquette term:

$$B(p) = \sigma^z_{p,1}\sigma^z_{p,2}\sigma^z_{p,3}\sigma^z_{p,4}$$

Hamiltonian:

$$H = -\sum_v A(v) - \sum_p B(p)$$

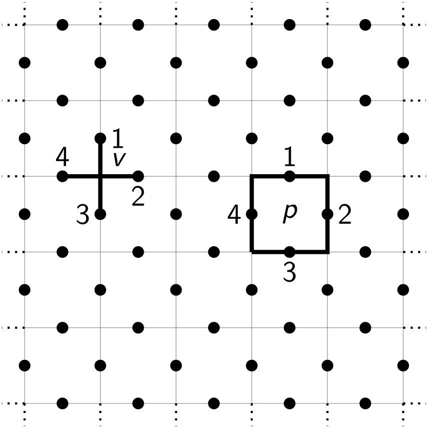


Figure: Lattice of Kitaev toric code

Kitaev Toric Code

Ground state:

$$|\xi\rangle = \prod_{v'} \frac{1}{\sqrt{2}}[I + A(v')]|00\cdots 0\rangle$$

Commutators:

$$[A, B] = [A, H] = [B, H] = 0$$

Eigenvalues:

$$A(v)|\xi\rangle = B(p)|\xi\rangle = +1$$

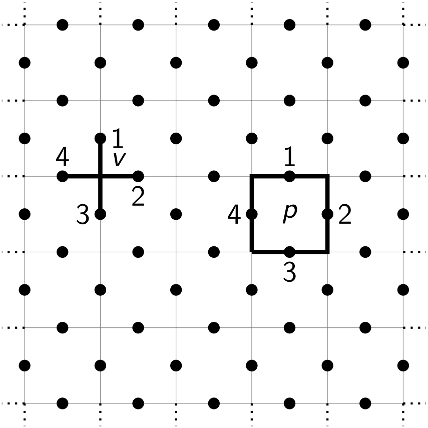


Figure: Lattice of Kitaev toric code

Creation and Annihilation of Anyons

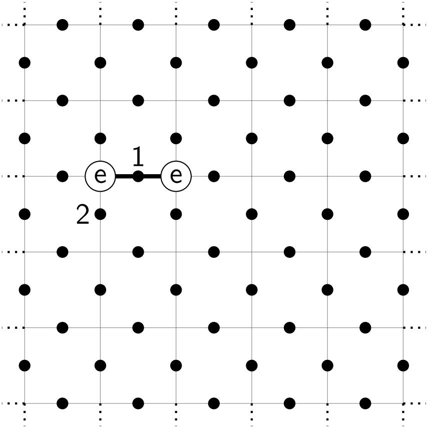
Creation of e anyon:

$$|e, e\rangle = \sigma_1^z |\xi\rangle$$

The reason:

$$A(v)|e, e\rangle = (-1)|e, e\rangle$$

Operator $A(v)$ detects e anyons at vertex v by eigenvalue -1.



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Creation and Annihilation of Anyons

Annihilation of e anyon:

$$\sigma_2^z|e,e\rangle = \sigma_2^z\sigma_1^z|\xi\rangle$$

This is because

$$A(v)\sigma_2^z\sigma_1^z|\xi\rangle = (+1)\sigma_2^z\sigma_1^z|\xi\rangle$$

No detection of e anyon on vertex v.

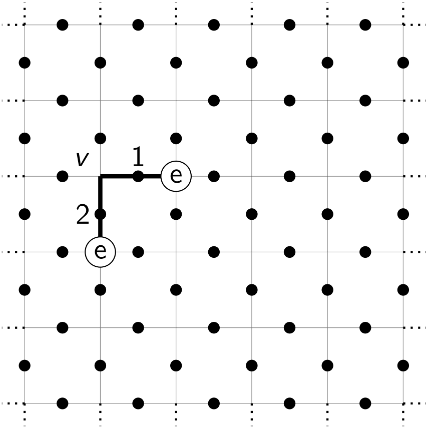


Figure: Lattice of Kitaev toric code

Creation and Annihilation of Anyons

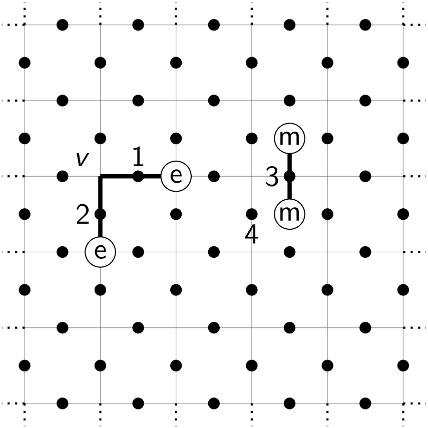
Creation of m anyon:

$$|m, m\rangle = \sigma_3^x |\xi\rangle$$

The reason:

$$B(p)|m, m\rangle = (-1)|m, m\rangle$$

Operator $B(p)$ detects e anyons at plaquette p by eigenvalue -1.



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Creation and Annihilation of Anyons

Annihilation of m anyon:

$$\sigma_4^x|m, m\rangle = \sigma_4^x\sigma_3^x|\xi\rangle$$

This is because

$$B(p)\sigma_4^x\sigma_3^x|\xi\rangle = (+1)\sigma_4^x\sigma_3^x|\xi\rangle$$

No detection of *m* anyon on plaquette *p*.

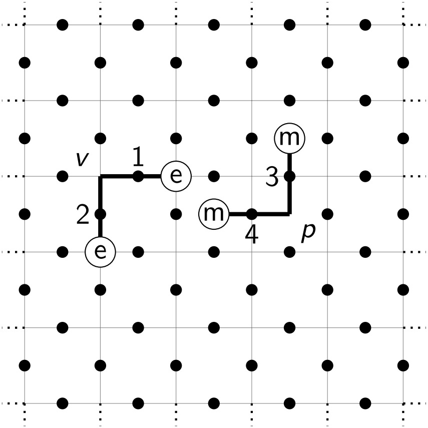


Figure: Lattice of Kitaev toric code

Creation and Annihilation of Anyons

Creation of ϵ anyon:

$$|\epsilon, \epsilon\rangle = \sigma^z \sigma^x |\xi\rangle$$

The reason:

$$A(v)|\epsilon, \epsilon\rangle = B(p)|\epsilon, \epsilon\rangle = (-1)|\epsilon, \epsilon\rangle$$

Both operator $A(v)$ and $B(p)$ detects ϵ anyons by eigenvalue -1.

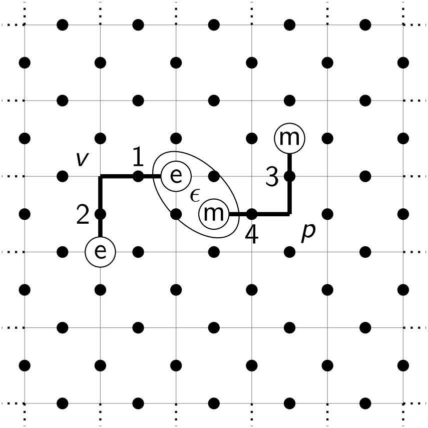


Figure: Lattice of Kitaev toric code

Fusion:

Bring two anyons together and determines how they behave collectively.
No interactions need to take place.

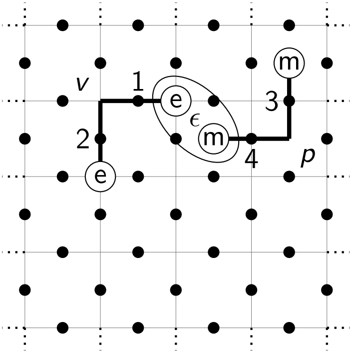
Fusion rule of toric code model:

$$e \times e = m \times m = \epsilon \times \epsilon = 1$$

$$e \times m = m \times e = \epsilon$$

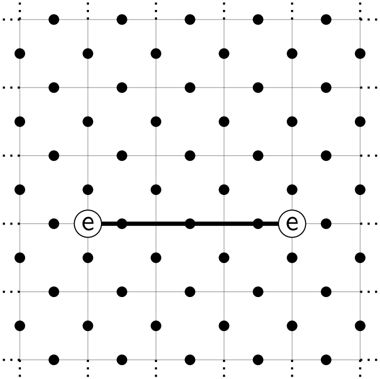
$$m \times \epsilon = \epsilon \times m = e$$

$$\epsilon \times e = e \times \epsilon = m$$



String on Lattice

Two anyons can be created anywhere, linked by a **string**. The state of the system is invariant with respect to deformations of the shape of the string as long as its endpoints remain fixed.



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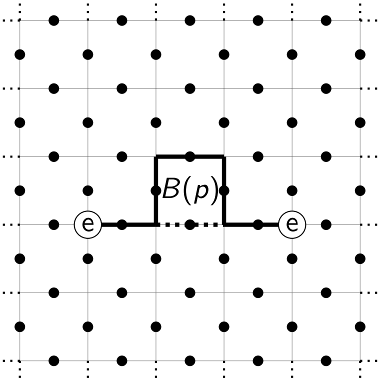

Figure: String on the lattice

Initial state:

$$|\psi\rangle = C_1 C_2 |\xi\rangle$$

Exchange of two e anyons:

$$C_{e\leftrightarrow e} = (\sigma_4^z \sigma_8^z)(\sigma_3^z \sigma_7^z)(\sigma_2^z \sigma_6^z)(\sigma_1^z \sigma_5^z)$$

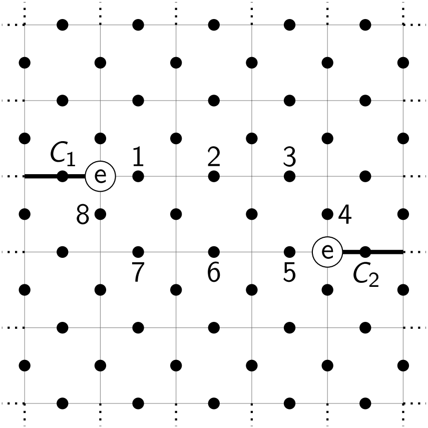

Figure: Exchange of e anyons

Statistics of Anyons

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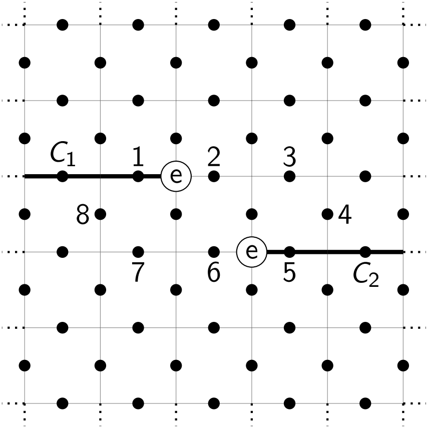

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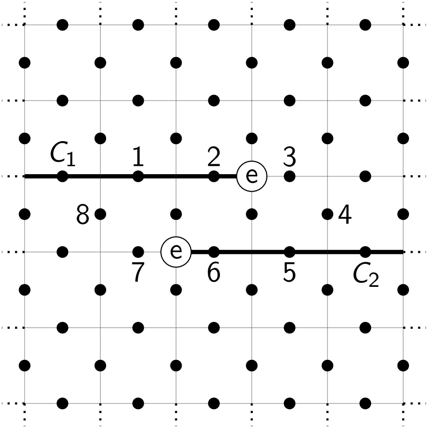

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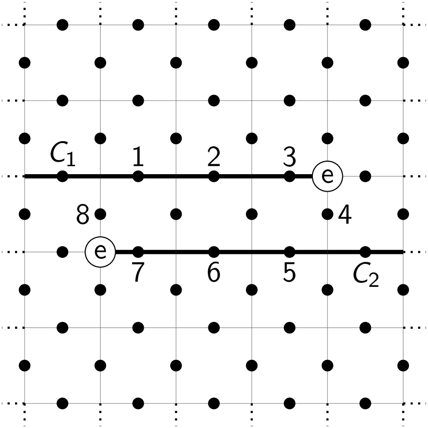

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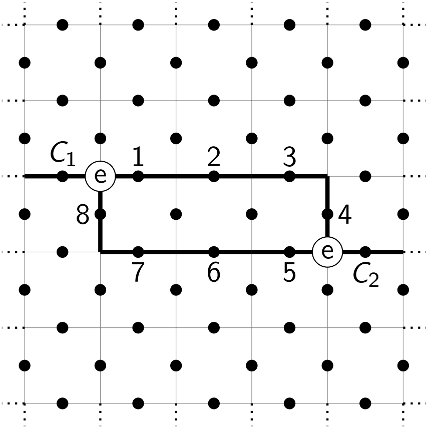

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Statistics of Anyons

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Exchange of two e anyons:

$$\begin{aligned} C_{e\leftrightarrow e} &= (\sigma_4^z \sigma_8^z)(\sigma_3^z \sigma_7^z)(\sigma_2^z \sigma_6^z)(\sigma_1^z \sigma_5^z) \\ &= B(p_1)B(p_2)B(p_3) \end{aligned}$$

Bosonic **mutual** statistics of e:

$$C_{e\leftrightarrow e}|\psi\rangle = |\psi\rangle$$

Similar for m anyons.

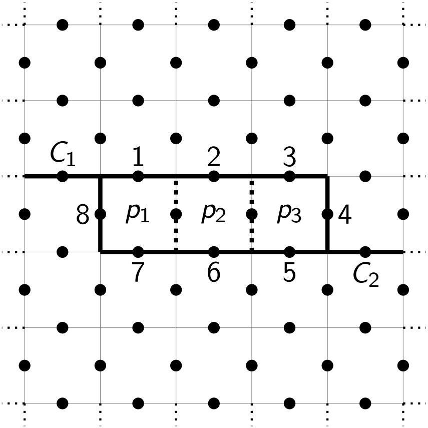

Figure: Exchange of e anyons

Statistics of Anyons

Exchange of two ϵ anyons:

$$\begin{aligned} C_{\epsilon\leftrightarrow\epsilon} &= {\color{blue}(\sigma_1^x\sigma_5^z\sigma_6^z\sigma_2^x)}{\color{red}(\sigma_2^z\sigma_4^x\sigma_3^x\sigma_1^z)} \\ &= (\sigma_1^x\sigma_2^x\sigma_4^x\sigma_3^x)(\sigma_5^z\sigma_6^z\sigma_2^z\sigma_1^z) \\ &= A(v)B(p) \end{aligned}$$

Mutual statistics of ϵ is also boson.

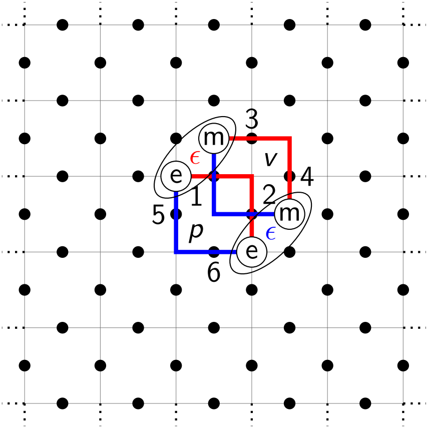

Figure: Exchange of ϵ anyons

Initial state:

$$|\psi\rangle = C_{\sigma^z} C_{\sigma^x} |\xi\rangle$$

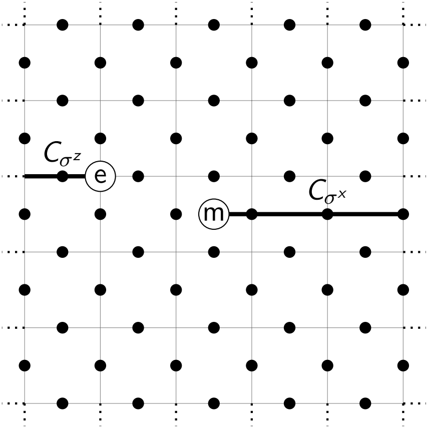

Figure: Braiding of e and m anyons

Statistics of Anyons

Initial state:

$$|\psi\rangle = C_{\sigma^z} C_{\sigma^x} |\xi\rangle$$

L_{σ^z} braiding e and m anyons, and

$$L_{\sigma^z} C_{\sigma^x} = -C_{\sigma^x} L_{\sigma^z}$$

Non-trivial phase arises:

$$L_{\sigma^z} |\psi\rangle = -|\psi\rangle$$

Similar for braiding ϵ with e or m.

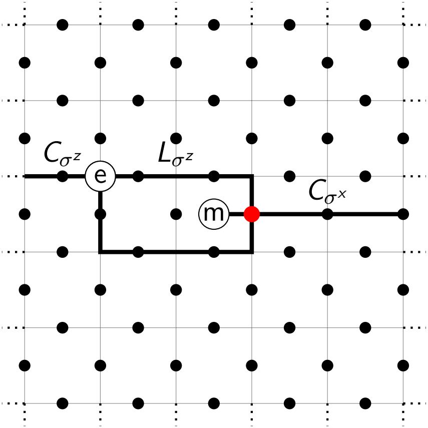

Figure: Braiding of e and m anyons

Statistics of Anyons

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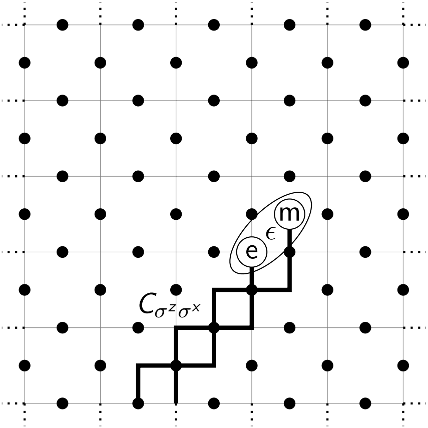


Figure: Self-rotation of ϵ anyons

Statistics of Anyons

Initial state:

$$|\psi\rangle = C_{\sigma^z\sigma^x}|\xi\rangle$$

Self-rotation of ϵ anyons L_{σ^z}, and

$$L_{\sigma^z} C_{\sigma^z\sigma^x} = -C_{\sigma^z\sigma^x} L_{\sigma^z}$$

Non-trivial phase arises:

$$L_{\sigma^z}|\psi\rangle = -|\psi\rangle$$

Indicates ϵ anyons are spin-$\frac{1}{2}$.

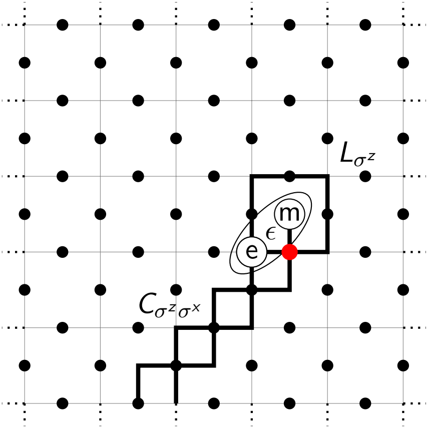


Figure: Self-rotation of ϵ anyons

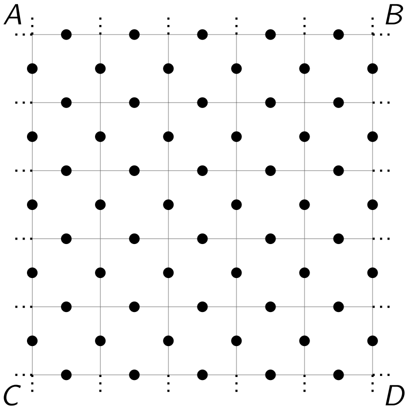
Encode Information in Toric Code

Ground state is used to store information. However, for now, the ground state is not degenerated, i.e. a 1-dimensional subspace.

Encode Information in Toric Code

Identify the corresponding sides to form a torus:

$$AB = CD \text{ and } AC = BD.$$



Encode Information in Toric Code

Unique property on torus:

$$\prod_{v} A(v) = 1 \text{ and } \prod_{p} B(p) = 1.$$

Only $2n^2 - 2$ operators A and B are independent.

Encode Information in Toric Code

Unique property on torus:

$$\prod_v A(v) = 1 \text{ and } \prod_p B(p) = 1.$$

Only $2n^2 - 2$ operators A and B are independent.
Define the **protected space** (which is the space of ground state):

$$\mathcal{L} = \{|\xi\rangle \in \mathcal{H} \mid A(v)|\xi\rangle = |\xi\rangle, B(p)|\xi\rangle = |\xi\rangle\}.$$

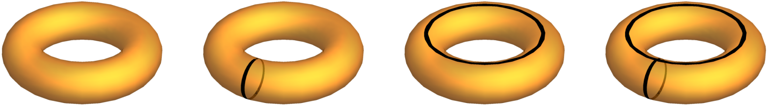
Now, the dimension is:

$$\dim \mathcal{L} = 2^{2n^2-(2n^2-2)} = 4.$$

Thus, we have two qubits.

Encode Information in Toric Code

Explicit description of four-fold degeneracy:
Create a pair of anyons and move them along a **non-contractible** path on the torus.

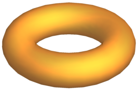


(a) $|\xi_1\rangle$ (b) $|\xi_2\rangle$ (c) $|\xi_3\rangle$ (d) $|\xi_4\rangle$

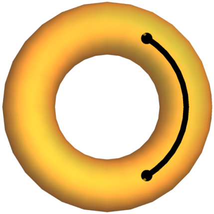
Figure: Four-fold degenerate ground state

Error Detection in Toric Code

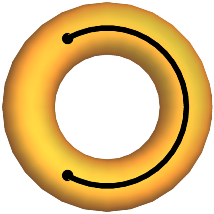
For example, the initial ground state is $|\xi_1\rangle$:



There are two kinds of errors might occur:



(a) Correctable



(b) Uncorrectable

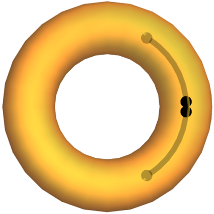
Figure: Four-fold degenerate ground state

Error Detection in Toric Code

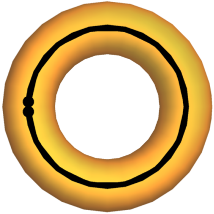
k-local operator

An operator that acts locally on at most k neighbouring subsystems.

Toric code protects qubits against $\lfloor \frac{n}{2} \rfloor$-local errors. The strategy is to annihilate the anyons through the **shortest** possible path on the geometry of the torus.



(a) Corrected to $|\xi_1\rangle$



(b) Error: become $|\xi_3\rangle$

Figure: Four-fold degenerate ground state

N-Fold Toric Code

The dimension of the ground state subspace is determined by the **genus** of the surface.

Genus:

Intuitively, genus is the number of handles of the surface.

On a compact orientable 2D surface of genus g, the ground state has dimension

$$\dim \mathcal{L} = 4^g.$$

The genus of torus is 1, so torus code has four-fold degeneracy. A n-fold torus has n handle, below is a 3-fold torus:

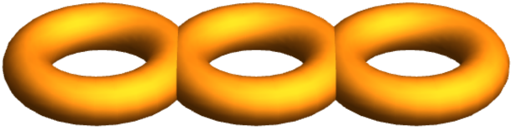


Figure: 3-Fold Torus

Algebraic Topology Point of View

Take B as an example:

- The set $\{B(p)\}$ generates the **1-boundary** group $B_1(T^2)$.
- Group of operators commuting with $\{B(p)\}$ is the **1-cycle** group $Z_1(T^2)$.
- $\mathcal{L}$ is the **1-homology** group

$$\mathcal{L} \cong H_1(T^2) = Z_1(T^2)/B_1(T^2) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2.$$

Other models

Kitaev honeycomb lattice model (Kitaev, 2006) supports both Abelian and **non-Abelian** anyons. **Ising anyon** and Majorana fermion arise in this model.

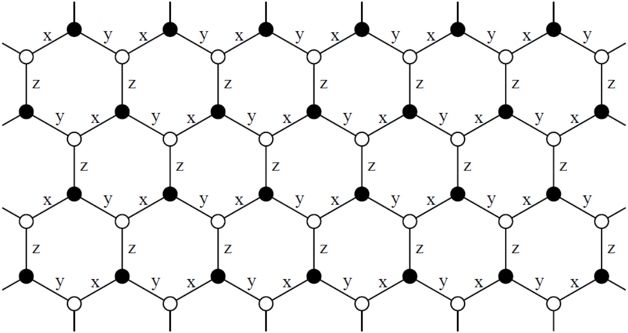


Figure: Kitaev honeycomb lattice model (Kitaev, 2006)

Section Summary

- Toric code is the simplest quantum double model that supports Abelian anyons.
- On toric code, Anyons have bosonic mutual statistics. On the other hand, non-trivial phase -1 arises when braiding two anyons.
- Toric code tolerates $\lfloor \frac{n}{2} \rfloor$-local errors.
- Non-trivial ground state subspace comes from the topological property of surface: **genus**.

Computation with Anyons

Abstract Anyon Model

Consider an abstract model of topological system, with finitely many species of anyons:

$$1(\text{vacuum}), a, \bar{a}, b, \bar{b}, c, \bar{c}, \cdots$$

Three processes described by worldline of anyons:

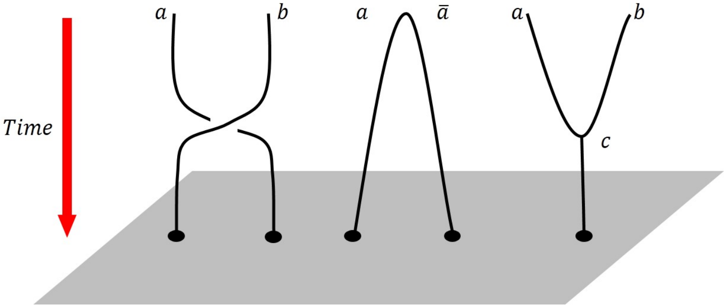


Figure: Braiding, creation and fusion of anyons

Fusion Rules

Fusion:

Bring two anyons together and determines how they behave collectively.
No interactions need to take place.

General fusion rule:

$$a \times b = N_{ab}^{c} c + N_{ab}^{d} d + \cdots$$

means that putting a and b together would give possible outcome of c, d and so on. Integer N_{ab}^{c} and N_{ab}^{d} indicate that there might be distinct mechanisms producing c and d. And here the order is not important:

$$a \times b = b \times a.$$

Fusion Rules

Abelian anyons have only a single fusion channel:

$$a \times b = c.$$

Non-Abelian anyons:

$$a \times b = N_{ab}^{c_1} c_1 + N_{ab}^{c_2} c_2 + \cdots \text{ where } \sum_{c_i} N_{ab}^{c_i} > 1,$$

which is due to the existence of **non-trivial evolution** between non-Abelian anyons.

Fusion Matrix

When we fuse several anyons, we are free to choose the ordering in which the basic fusion processes take place.

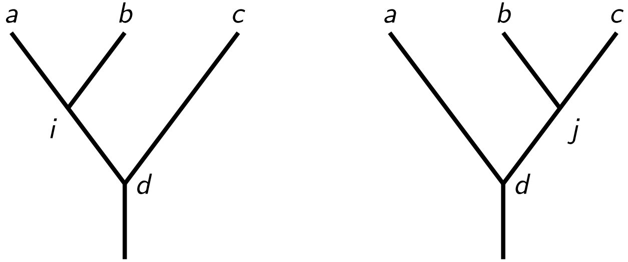


Figure: Conversion between different in-between state

F^d_{abc} is called the **fusion matrix** or F matrix.

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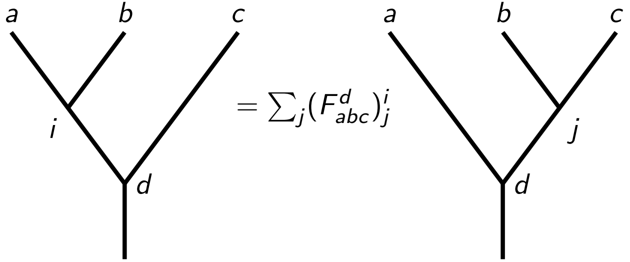

Figure: Conversion between different in-between state

F^d_{abc} is called the **fusion matrix** or F matrix.

Exchange Matrix

According to the statistics of anyons, the exchange of two anyons would give rise to a phase factor. Set $(R_{ab})^c$ equal to the phase factor acquired by the wave function when exchange a and b getting c.

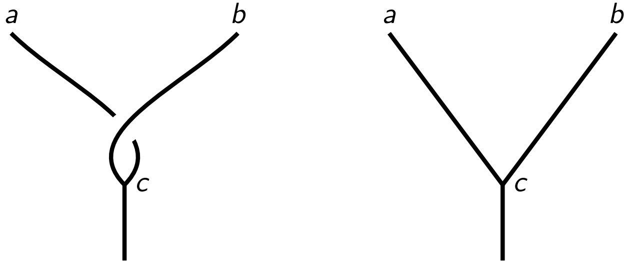

Figure: Extra phase factor due to exchange of anyons

Note that $(R_{ab})^c$ is simply a number while R_{ab} is a matrix whose diagonal elements are $(R_{ab})^c$, called **exchange matrix** or R matrix.

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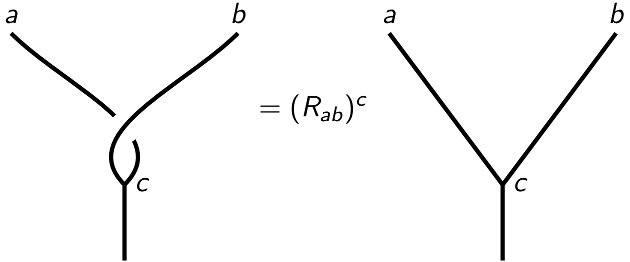

Figure: Extra phase factor due to exchange of anyons

Note that $(R_{ab})^c$ is simply a number while R_{ab} is a matrix whose diagonal elements are $(R_{ab})^c$, called **exchange matrix** or R matrix.

Braiding Matrix

Because there are multiple fusion outcomes in the braiding process, we need **braiding matrix** (B matrix) B_{ab}.

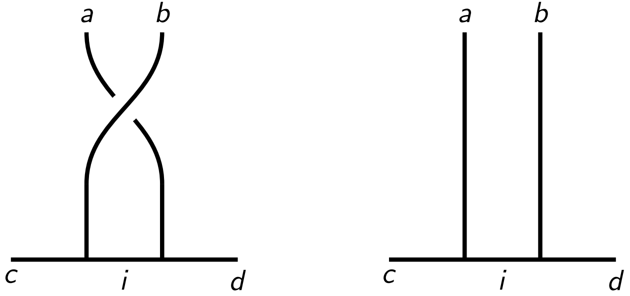

Figure: Braiding matrix of anyons

It can be proved that

$$B_{ab} = (F^d_{acb})^{-1} R_{ab} F^d_{acb} \text{ or concisely } B = F^{-1} R F.$$

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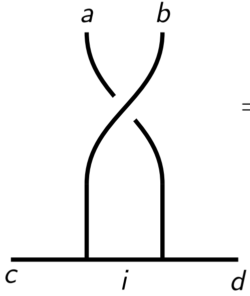

$$= \sum_j (B_{ab})^i_j$$

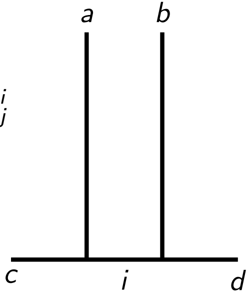

Figure: Braiding matrix of anyons

It can be proved that

$$B_{ab} = (F^d_{acb})^{-1} R_{ab} F^d_{acb} \text{ or concisely } B = F^{-1}RF.$$

Identities of F and R Matrices

Pentagon identities:

$$(F^5_{12c})^d_a (F^5_{a34})^c_b = \sum_e (F^d_{234})^c_e (F^5_{1e4})^d_b (F^b_{123})^e_a.$$

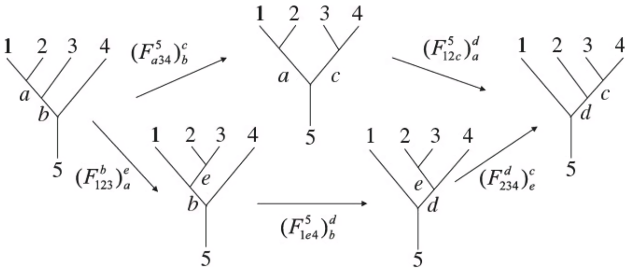

Figure: Pentagon identity (Pachos, 2012)

Identities of F and R Matrices

Hexagon identities:

$$R_{13}^{c}(F_{213}^{4})_{a}^{c}R_{12}^{a}=\sum_{b}(F_{231}^{4})_{b}^{c}R_{1b}^{4}(F_{123}^{4})_{a}^{b}.$$

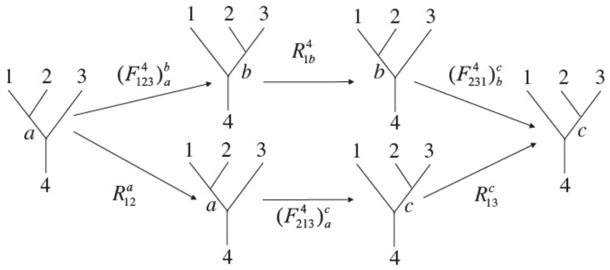

Figure: Hexagon identity (Pachos, 2012)

Anyonic quantum computation

Quantum circuit model	Anyonic model
State initialization	Create and arrange anyons
Quantum gates	Braid anyons
Measurement	Detect anyonic charge

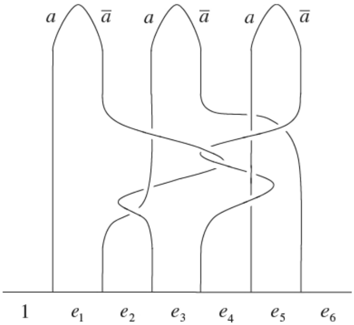

Figure: Skectch of anyonic computation (Pachos, 2012)

Example: Ising Anyons

Anyon types of Ising anyon model:

$$1 \text{ (vacuum)}, \sigma \text{ (non-Abelian anyon)}, \psi \text{ (fermion)}.$$

Fusion rules:

$$\begin{aligned} &\sigma \times \sigma = 1 + \psi,\ \sigma \times \psi = \sigma,\ \psi \times \psi = 1, \\ &\sigma \times 1 = \sigma,\ \psi \times 1 = \psi. \end{aligned}$$

F and R matrices:

$$F^{\sigma}_{\sigma\sigma\sigma} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},\ R_{\sigma\sigma} = e^{-i\pi/8} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}.$$

Example: Ising Anyons

$$|0\rangle_{\text{Logical}} = |(\sigma,\sigma) \to 1\rangle \text{ and } |1\rangle_{\text{Logical}} = |(\sigma,\sigma) \to \psi\rangle$$

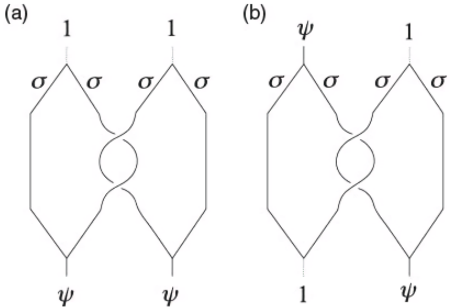

Figure: NOT gate (Pachos, 2012)

Example: Ising Anyons

$$NOT = (F^{-1}RF)(F^{-1}RF) = F^{-1}R^2F = e^{-i\pi/4}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

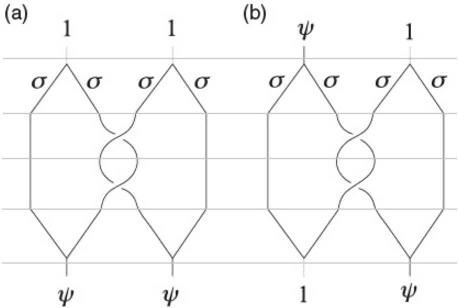

Figure: NOT gate (Pachos, 2012)

Section Summary

- The process of anyonic computation is creation, braiding and detection of anyons.
- Ising anyons model is not universal since its F and R matrices cannot span SU(2). Other model such as **Fibonacci anyons** (Trebst, 2008) can.

Application

Jones Polynomial

Topological invariant:

Topological invariant is a property of a topological space which is invariant under continuous deformation.

Same topological space $\Rightarrow$ **Same topological invariant**

Different topological invariant $\Rightarrow$ **Different topological space**

The investigation of topological invariant is essential in many fields of study.

Jones Polynomial

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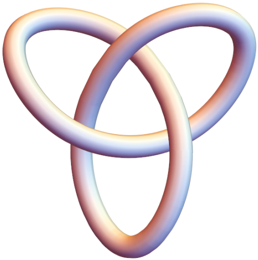
The investigation of topological invariant is essential in many fields of study.

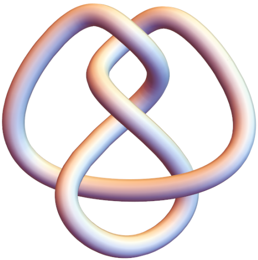
Jones polynomial:

Jones polynomial is a topological invariant of **knot** or **link**.

Jones Polynomial



(a) $-A^{-4}+A^{-3}+A^{-1}$



(b) $A^{-2}-A^{-1}+1-A+A^{2}$

Figure: Two different knots with different Jones polynomials

Jones polynomials are found to be important in many place: topological quantum field theory, DNA reconstruction, statistical physics, ...

Jones Polynomial

Unfortunately, the best known classical algorithm for the evaluation of Jones polynomials requires **exponential** resources.
Using anyons, computation Jones polynomials is quite efficient and straight-forward, like an **analogue computer**.

Outlook

Outlook

- TQC is more suitable to be described in topological quantum field theory. The TQFT formalism of TQC is quite mature by now (from Chern-Simons QFT).
- Although seems mysterious and theoretical, experiments have been intensively carried out by physicists. Focus of experimental realisation lies mainly on fractional quantum Hall effect.
- Information theory can apply. For example, topological entropy can be discussed in such a topological system.

Thank You!